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A DEPARTURE TIME CHOICE MODEL FOR DYNAMIC ASSIGNMENT ON INTERURBAN NETWORKS

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INTRODUCTION

The increasing interest for time-varying tolling policies in transportation networks has highlighted the need for dynamic traffic assignment models in which users are allowed to choose both their route from origin to destination and their departure time. In the case of a single route with a bounded capacity, Vickrey (1969) derived the departure rate as the outcome of individual cost minimization. In Vickrey's model, for a given departure time, individual costs are modelled as continuous, unimodal and convex functions of the schedule delay, i.e. as a penalty for early or late arrivals at destination. Since then a number of extensions have been proposed, notably to deal with complex networks, multiple origins and multiple destinations. Most of those extensions (see for instance Heydecker and Addison, 2005 or de Palma and Lindsey, 2006) are tailored for studying equilibrium in metropolitan networks, in which commuting travel patterns are assumed: the preferred arrival time to a destination is often taken from a normal distribution around a peak hour, and individual schedule costs essentially follow Vickrey's model.

Unfortunately, those assumptions do not hold in the case of large interurban networks. In this case, the economic behaviour of a trip-maker with respect to its preferred arrival time varies widely from one to the other. Some travellers may want to be at destination early in the morning, some others late in the evening, while some others require to be at destination absolutely before a given hour, etc. Moreover, a significant percentage of the travellers may wish to avoid peak days, by rescheduling their departure day to the day before or to the day after, if incited to do so.

This article presents extensions made to a dynamic traffic assignment model to encompass those needs. It has the following outline. Section 1 contains a brief review of the literature. It helps in highlighting the distinctive features of our model, which is introduced in section 2. The two main algorithms used to compute a user equilibrium with departure choice are provided in section 3. Some results from numerical studies are presented in section 4, before concluding.

1. LITERATURE REVIEW

The seminal paper on trip scheduling (Vickrey, 1969) essentially deals with morning commute. A fixed number of commuters is considered, travelling from a single origin to a single destination, through a single route. Congestion occurs at a bottleneck of bounded capacity. Users are modelled as microeconomic agents minimizing a cost function that depends on travel time and schedule delay. In the simplest version of the model, Vickrey considers homogeneous users that all have the same preferred arrival time and the same cost function, linear in travel time and piece wise linear convex in schedule delay. The main results from this model are recalled in

subsection 1.1. Since then, several extensions have been proposed. Some of them are the topic of subsection 1.2. However, as shown in subsection 1.3, some empirical evidences from field data indicate that those various extensions may not fulfil the requirements of departure choice modelling in interurban networks.

1.1. Main results from the bottleneck model

A set of commuters wish to reach a CBD accessible by one route with bounded capacity. Each user is characterized by a preferred arrival instant h^* , and assesses the decision of departing at an instant h using a cost function of the form:

$$G(h) = \alpha t(h) + \beta (d(h))^- + \gamma (d(h))^+ \quad (1)$$

where:

- $t(h)$ is the travel time when departing at h ,
- $d(h) = h + t(h) - h^*$ is the delay incurred at arrival when departing at h ,
- α is the value of time of the commuter,
- β [resp. γ] it the marginal cost of arriving earlier [resp. later] than preferred,
- $()^-$ and $()^+$ stand for the positive and negative part of the delay.

There are two standard ways of describing the set of users. Either one considers a finite number of categories of commuters, differing by their h^* (and also possibly their value of time, value of arriving late or early); or one considers that users have preferred arrival instants distributed among a set of possible values. In the later case, the “S-shape” assumption is made: there is a single interval during which the density of commuters exceeds capacity. This assumption makes the model analytically tractable, and induces a travel time pattern similar to the one with a unique h^* shared by all commuters.

A typical equilibrium situation is depicted in Fig.1 for a bottleneck of capacity K . The set of users is modelled by a cumulative distribution X^* over the set of their preferred arrival instants. Their choices of departure instants are given by the cumulative distribution X_+ . The bottleneck model allows one to compute the cumulative distribution X_- of users at the exit of the bottleneck.

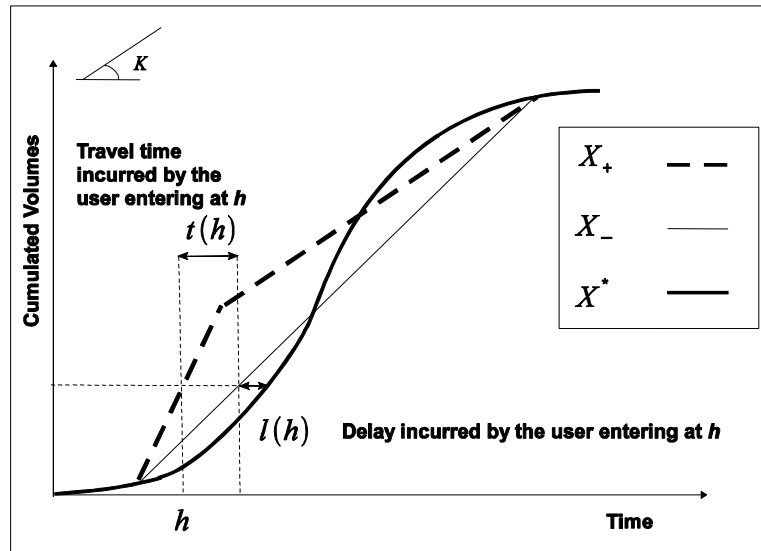


Figure 1: The bottleneck model.

Fig.1 can be interpreted as such: the horizontal difference between X_+ and X_- (i.e. $t(h)$) gives the amount of time needed to traverse the bottleneck, when entering the bottleneck at instant h . The horizontal difference between X_- and X^* (i.e. $l(h)$) gives the delay at arrival. The traversal time $t(h)$ is a piecewise function with only two admissible slopes and a single maximum. It increases at the beginning of the congestion period, when users are arriving earlier than preferred. When $t(h)$ is decreasing, users are arriving later than preferred. Note that the simple form of the delay cost function (i.e. the two last terms of Eq.1) implies the piece wise linear shape of the travel time function. Under more general assumptions, it would be smoother.

1.2. Extensions to the bottleneck model

Assumptions made in Vickrey's model are indeed well tailored for morning commutes: the simple congestion model makes sense for trips toward a small CBD area; a unique preferred arrival time is adapted for work starting time; finally a convex schedule delay cost function does fit with job schedule constraints. Since then, the transportation community has investigated the field in two main directions. Some works, mainly from transport economists, have focused on users heterogeneity. Others have proposed extensions to whole networks.

Departure time choice with users heterogeneity

Heterogeneity in preferred arrival times can be addressed either in discretely fashion, by allowing only a finite set of preferred arrival times, or continuously using a distribution. The finite case has been studied extensively by Lindsey (2004) while the continuous case was first treated by Hendrickson and Kocur (1981). Heterogeneity pertaining to the costs of travel time and of schedule delay has been studied, among others, by (de Palma *et al.*,1993) and (Van Der Zijpp and Koolstra, 2002). Other extensions include the modelling of stochastic demand and capacity, multiple routes or elastic demand (see (de Palma *et al.*,1998) for a review). When users are at equilibrium, the bottleneck model predicts a congestion pattern with a single peak in travel time. In the numerous extensions, the resulting congestion patterns are very similar to the homogeneous case. When considering a finite number of preferred

arrival instants, there is a limited number of peaks in travel time (at most one per preferred arrival instant) and a spontaneous segregation among users is observed. Commuters with different preferred arrival instants depart at different instants (Lindsey, 2004). The case where users' preferences are distributed over an interval has received less attention. Papers in this line mainly considered "S-shape" distribution (See Smith, 1984 and Daganzo, 1985). As exposed in section 1.1 this case is practically equivalent to the one with a single preferred arrival time and produces exactly the same travel time pattern.

Departure time choice on networks

The bottleneck model has been extended to networks, in an attempt to produce operational planning models. The computation of the user equilibrium in such a context is known as the dynamical traffic assignment problem with departure time choice. Friesz *et al* (1993) first proposed a formulation of the user equilibrium with both route and departure time choice. His model considers users spread between origin-destination (o-d) pairs, with a unique preferred arrival instant by o-d. Since then most of the models proposed in the literature rely on Friesz's original paradigm (see for instance Wie *et al*, 2001). Rather than focusing on users heterogeneity, this part of the literature has made considerable efforts to improve congestion representation by integrating sophisticated traffic models.

1.3. Empirical evidences and their practical implications for departure time choice modelling.

Two assumptions underlie the bottleneck model and its various extensions: (i) preferred arrival times are taken from a discrete set of values and (ii) delay cost functions are convex. As discussed in this subsection, those two assumptions appear not to be appropriate means of modelling economic preferences of inter-urban trip makers.

Let us first have a look at the travel time patterns and flow rates observed in interurban trips. Fig.1(a) shows the variations of travel time between Lyon and Montpellier, two cities of southern France, on a holiday departure day. The pattern is quite far from the single peak predicted by the bottleneck model: at least two peak periods can be observed, along with significant variations elsewhere. The flow rate on the same o-d pair is plotted in Fig.1(b). One can observe significant flow rates during the whole day. This is clearly not consistent with a single preferred arrival instant.

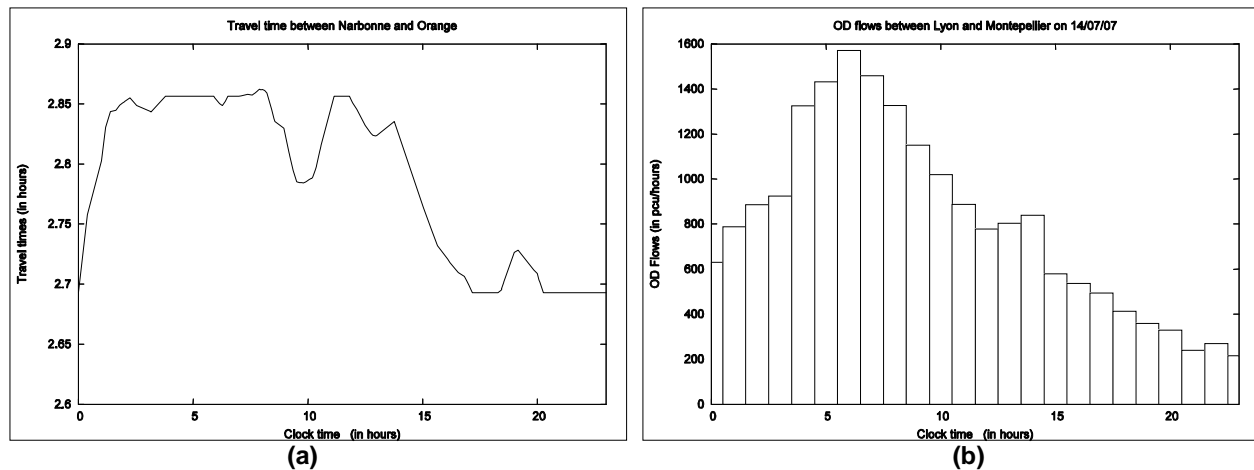


Figure 1. (a) Travel time pattern between Narbonne and Orange on the 14th of July 2007. (b) o-d flows between Narbonne and Orange (courtesy of ASF).

Another interesting point is the diversity of inter urban travellers. As opposed to the morning peak where the road traffic is mainly composed of commuters, inter urban trips have a wide variety of purposes, inducing significant differences in value of time and delay cost functions. In the same order of idea, a significant part of the traffic is composed of heavy vehicles, which has important consequences on congestion modelling.

Finally the results of a survey organized in 2008 by three French motorway operators show that, during summer holidays, trip makers can be divided into two categories: some can absolutely not afford arriving later than scheduled (e.g. because they need catching the key for their rental). Some others are far less constrained at arrival and are even ready to reschedule their departure day, if they can benefit from lower congestion and toll fares. This last point is especially interesting as it shows that in inter urban context the convexity of delay cost functions can no longer be assumed. Indeed in this case the cost of the delay does not necessarily decrease as the arrival time gets closer to the preferred schedule. A traveller considering to leave one day in advance to avoid traffic jams will not necessarily consider arriving at 2 a.m. a better option.

To sum it up, empirical observations show that, for inter urban trips, a departure time choice model should differ from the classical “bottleneck-like” approach, with respect to the three following requirements:

1. A high level of heterogeneity regarding both preferred schedules (several preferred arrival times per o-d pair) and economic characteristics (value of times and schedule delay cost functions) is required.
2. Multi class congestion modelling is to be considered.
3. Users should be able to choose their day of departure as well as the time of the day.

2. THE DYNAMIC ASSIGNMENT PROBLEM WITH DEPARTURE TIME CHOICE.

In order to meet the modelling challenges exposed just below, we present an extension based on the LADTA ¹ model proposed by Leurent (2003). The dynamic

assignment with departure time choice is modelled in a supply demand framework where the supply is a network of bottlenecks and the demand a set of microeconomic agents. Those agents are characterized by economic preferences (value of time and delay cost function), temporal preferences (preferred arrival time) and physical characteristics (vehicle type). The network is subject to congestion and hence to a given demand the supply model associates a set of arc traversal time functions. Similarly the demand reacts to the supply by adjusting the time varying flows at the entrance of each route of the network according to the level of congestion. The existence of solutions of such assignment problems has been established recently by Meunier and Wagner (2009) in a continuous game theory framework.

In its original version, LADTA deals with multi-class dynamic traffic assignment considering a fixed demand: the demand is expressed as a dynamic o-d matrix that solely depends on departure instants from the origins. In the sequel, this problem is called the *user optimum assignment with fixed departure time*. It is stated as a fixed point problem in subsection 2.1, along with some helpful notations. Next, subsection 2.2 states the *user optimum assignment with departure time choice* problem in a similar way.

2.1. User optimum assignment with fixed departure time

Let us model a road network as a directed graph $G = (N, A)$, where N is a finite set of nodes and $A \subseteq (N \times N)$ is the set of arcs in G . A route r in G is a finite, non empty, sequence of connected arcs. R denotes the set of routes in G . Users of the transportation network are modelled as a continuum of microeconomic agents, where each user belongs to a user class u . U denotes the set of user classes. To each route r in R is associated a route traversal time function $t_{r,u}(h)$, where h is a departure instant from the head node of the route, taken in a continuous set of instants H . Also, a route traversal cost function $c_{r,u}(h)$ is associated to each route r in R . If $i = (o, d), o \neq d$ is a distinguished pair of nodes in $N \times N$, then $R_i \subseteq R$ denotes the set of routes starting at o and ending at d . $x_{i,u}(h)$ is the density of users wishing to depart from o at instant h , and $X_{i,u}(h) = \int_{h' < h} x_{i,u}(h') dh'$ is the cumulated distribution of such users.

Route choice

Given a fixed set I of o-d pairs, a demand $X_{I,U} = \{X_{i,u}\}$, and a set of route traversal cost functions $c_{R,U} = \{c_{r,u}\}$, a route choice RC is the definition of a set $X_{I,R,U} = \{X_{i,r,u}\}$ of distributions of users on routes per o-d pair such that, for all (i, h, u) triples, the two following equations hold:

$$\sum_{r \in R_i} X_{i,r,u}(h) = X_{i,u}(h) \quad (2)$$

$$x_{i,r,u}(h) \neq 0 \Rightarrow c_{r,u}(h) = \min_{r \in R_i} \{c_{r,u}(h)\} \quad (3)$$

Eq. (2) expresses that, at every departure instant, the demand of users of class u on the o-d pair i is distributed among the routes of this o-d pair. Eq. (3) expresses that only routes of minimal cost are chosen. Using a compact notation:

$$X_{I,R,U} = \text{RC}(X_{I,U}, c_{R,U})$$

Traffic flowing

Knowing a distribution $X_{I,R,U}$ of users on routes, a traffic flowing TF is a function that returns a set of routes traversal cost and time functions. Using a compact notation:

$$(c_{R,U}, t_{R,U}) = \text{TF}(X_{I,R,U})$$

Details concerning the computation of a traffic flowing are omitted here. The reader interested by the actual implementation of traffic flowing in LADTA can refer to (Aguiléra and Leurent, 2009).

User optimum assignment with fixed departure time

Given a demand $X_{I,U}$, a user optimum assignment with fixed departure time UOA($X_{I,U}$) is a route choice $\tilde{X}_{I,R,U}$ such that:

$$\text{UOA}(X_{I,U}) = \tilde{X}_{I,R,U} \in \{\text{RC}(X_{I,U}, \text{TF}(\tilde{X}_{I,R,U}))\} \quad (4)$$

2.2. User optimum assignment with departure time choice

Delay cost function

A delay cost function D is a positive and continuous mapping between delays and cost units, such that $D(d) \rightarrow +\infty$ when $|d| \rightarrow +\infty$. In practice, D is often such that $D(0) = 0$, but this is not a requirement.

Optimal routes and departure instants

When travelling from o to d , if a user $x_{i,u}$ wants to reach the destination d at a given arrival instant h^* , following a given route r in R_i , he has to choose a departure instant h such that $h + t_{r,u}(h) - h^* = 0$. More generally, if D_u is a delay cost function associated to the user class u , and if $x_{i,u}$ wishes to minimize its total traversal cost, he has to choose a departure instant h that minimizes

$$G_{u,h^*}(r, h) = c_{r,u}(h) + D_u(h + t_{r,u}(h) - h^*)$$

Departure choice

For a given arrival instant h^* , let $x_{i,u}^*(h^*)$ be the density of users of class u on the o - d pair $i = (o, d)$ wishing to minimize their total traversal cost from o to d . Let $\Gamma_{i,u}(h^*)$ the subset of $R_i \times H$ where the total traversal cost G_{u,h^*} reaches its minimum. Then any set of density functions χ_{i,u,h^*} such that :

$$\int_{(R,H)} \chi_{i,u,h^*} = \int_{(r,h) \in \Gamma_{i,u}(h^*)} \chi_{i,u,h^*} = x_{i,u}^*(h^*)$$

defines a departure choice $X_{I,R,U}$, using the demand density functions defined by $x_{i,r,u} = \int_{h^*} \chi_{i,u,h^*}(r, \bullet)$. Using a compact notation:

$$X_{I,R,U} = \text{DC}(X_{I,U}^*, (c_{R,U}, t_{R,U})) \quad (5)$$

User optimum assignment with departure time choice

Given a demand $X_{I,U}^*$, a user optimum assignment with departure time choice

$\text{UOA}^*(X_{I,U}^*)$ is a route choice $\tilde{X}_{I,R,U}$ such that:

$$\text{UOA}^*(X_{I,U}^*) = \tilde{X}_{I,R,U} \in \{\text{DC}(X_{I,U}^*, \text{TF}(\tilde{X}_{I,R,U}))\} \quad (6)$$

3. ALGORITHMS.

For a given o-d pair, an exact computation of the departure choice as defined by Eq.5 involves managing the possibly infinite set of routes on this o-d (by definition, each route is finite, but looping routes are not excluded). However, it appears reasonable in practice to concentrate on a finite subset of efficient routes. The *departure choice* problem is then casted down to the *departure time choice on a single route*, for which an algorithm is detailed in subsection 3.1. An algorithm for the combined route choice and departure time choice is then proposed in subsection 3.2.

3.1. Departure time choice on a single route

Let r be a route and assume we are given the traversal time t_r and cost c_r for every departure instant h on that route. Now suppose that a demand on that route is expressed as a distribution X_r^* of users, as a function of preferred arrival instants h^* . All users in X_r^* share a same delay cost function. This section presents an algorithm, called DTCR, which computes the distribution $X_r(h)$ at the entrance of the route for every departure instant h and consistent with the pair $(t_r; c_r)$. The basic idea of the algorithm is as follows: for each h^* of a sample of preferred arrival instants, find the set of departure instants that lead to a total traversal cost that is “close” to the minimum. Then, spread uniformly over this set the volume of users with preferred arrival time close to h^* .

Algorithm DTCR

Inputs

- c_r (resp. t_r) : the route traversal cost (resp. time) function of r .
- $X_r^*(h^*) = \int_{h < h^*} x_r^*(h) dh$: a demand with departure time choice on r , such that $x_r^*(h^*) = 0$ for every h^* outside a range $[h_a^*; h_b^*]$ of preferred arrival instants.
- D : a delay cost function.
- δG : a small positive number.
- n : a positive integer.

Output

- $X_r(h)$: a cumulated distribution of users for every departure instant from the origin of r

Begin

Let h_k^* , $k = 0 \dots n$ be a discrete sampling of $[h_a^*; h_b^*]$.

$X_r \leftarrow 0$

For $k = 1 \dots n$

1. Let $\Delta X_{r,k}^*$ be the number of users in the interval $[h_{k-1}^*; h_k^*]$

$$\Delta X_{r,k}^* \leftarrow \int_{h_{k-1}^*}^{h_k^*} x_r^*(h) dh$$

2. Let h^* be the midpoint of $[h_{k-1}^*; h_k^*]$

$$h^* = \frac{h_k^* + h_{k-1}^*}{2}$$

3. Compute the total cost function G such that :

$$G(h) = c_r(h) + D(h + t_r(h) - h^*)$$

4. Let g be the minimum value of G

$$g \leftarrow \min_{h \in H} \{G(h)\}$$

5. Compute the set H^* where G is close to its minimum

$$H^* \leftarrow \{h \in H, G(h) - g < \delta G\}$$

6. Compute the function $\Delta X_{r,k}(h)$ such that the users $\Delta X_{r,k}^*$ are uniformly distributed on H^* .

7. Add $\Delta X_{r,k}$ to X_r

$$X_r \leftarrow X_r + \Delta X_{r,k}$$

End For

End

3.2. Combined route choice and departure time choice

Algorithm DTCR solves a departure choice problem with no route choice and a fixed supply. We are now going to take advantage of it in order to solve the combined route and departure time choice problem. At each iteration k of the algorithm, we are given a fixed departure time o-d matrix $X_{I,U}^{[k-1]}$, and we first find the user optimal assignment with fixed departure time, thus obtaining for every instants and on every o-d pair an optimal route together with its traversal time and cost $(c_{i,u}^{[k]}, t_{i,u}^{[k]})$. Using DTCR it is then possible to compute the optimal departure time decision of the users with respect to $(c_{i,u}^{[k]}, t_{i,u}^{[k]})$ and to derive a new fixed departure time o-d matrix $X_{I,U}^{[k]}$.

Algorithm UOA*

Inputs

- $X_{I,U}^*$: a set of cumulated distributions over $[h_a^*; h_b^*]$

Output

- $X_{I,R,U}$: a user optimum assignment with departure time choice

Begin

$$X_{I,U}^{[0]} \leftarrow \text{DTCR}(c_{I,U}^{[0]}, t_{I,U}^{[0]})$$

$$k \leftarrow 1$$

Do

1. Compute a user optimum assignment with fixed departure time from the demand $X_{I,U}^{[k-1]}$

- $$(X_{I,R,U}^{[k]}, c_{I,U}^{[k]}, t_{I,U}^{[k]}) \leftarrow \text{UOA}(X_{I,U}^{[k-1]})$$
2. For each o-d pair i and user class u , compute the auxiliary demand $Y_{i,u}^{[k]}$ using $(c_{i,u}^{[k]}, t_{i,u}^{[k]})$ as inputs of the DTCR algorithm

$$Y_{i,u}^{[k]} \leftarrow \text{DTCR}(c_{i,u}^{[k]}, t_{i,u}^{[k]})$$
 3. Increase k and compute w_k

$$k \leftarrow k + 1$$

$$w_k = 1/k$$
 4. Compute $X_{I,U}^{[k]}$

$$X_{I,U}^{[k]} \leftarrow (1 - w_k)X_{I,U}^{[k-1]} + w_k Y_{I,U}^{[k]}$$
- While** $X_{I,U}^{[k]}$ differs significantly from $X_{I,U}^{[k-1]}$
- $$X_{I,R,U} \leftarrow \text{UOA}(X_{I,U}^{[k]})$$

End

4. NUMERICAL EXAMPLES.

The two algorithms exposed in section 3 have been implemented within the LTK ², an implementation of LADTA that allow for computing dynamic traffic assignments on large networks (Aguiléra and Leurent, 2009). In this section, the results coming from two test cases are presented. The first one, called SR91, is a small network with one origin, one destination, two routes and two user classes. Albeit simple, it illustrates combined route and departure time choice. It is also a well known case study for transportation economists and as such most of the data regarding both scheduling cost functions and value of time are available in the literature. The second example, called VDR, is an application to a large network.

It has to be clear to the reader that what follows in subsections 4.1 and 4.2 has to be understood as numerical experiments. Their sole purpose is illustrative, and the conclusions, figures and charts presented therein have no particular meaning outside the scope of this paper.

4.1. The SR91 example

The State Route 91 is located in the orange county (California, USA) and was faced to an important congestion problem at the beginning of the 90's. The road is connecting a residential zone to a labour pool. Before 1995, it had a capacity of 8000 pcu/h, and four lanes in each direction. In 1995 two lanes were added in each direction. The two additional lanes are equipped with time varying tolls, in both directions. The tolls are used as a congestion management tool to alleviate traffic on peak hours by spreading demand. Yet this management is complex: a variation in the toll fare can induce both trip rescheduling and rerouting. Commuters are highly heterogeneous regarding their arrival time preference (Sautter, 2007), so the travel time pattern at equilibrium is likely to have a non trivial form.

User classes

Two user classes, subscripted r and p , are considered. Both of them have a delay cost function D under the classic V-shape form:

$$D(d) = \beta \cdot (d)^- + \gamma \cdot (d)^+$$

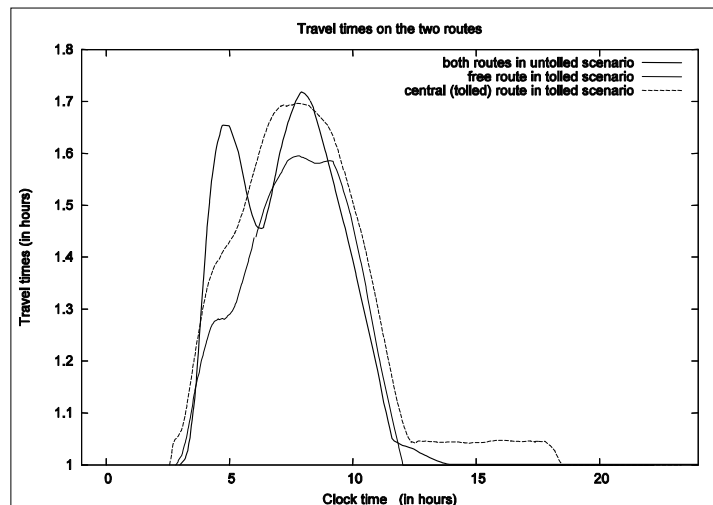
They differ only by their value of time α , their unit cost of arriving late, proportional to β and their unit cost of arriving early, proportional to γ . For user class p , α_p is taken from Lam et Small (2001) study on SR91. γ_p and β_p are the ratios obtained by Small (1982) in its study for San Francisco. In dollars, the values are $\alpha_p = 22.87$, $\gamma_p = 38.12$ and $\beta_p = 12,20$. For user class r , along the line of the technical report from Sautter (2007), it is assumed that the ratios β_r / α_r and γ_r / α_r are the same, and that $\alpha_r = 2\alpha_p$.

Demand and supply

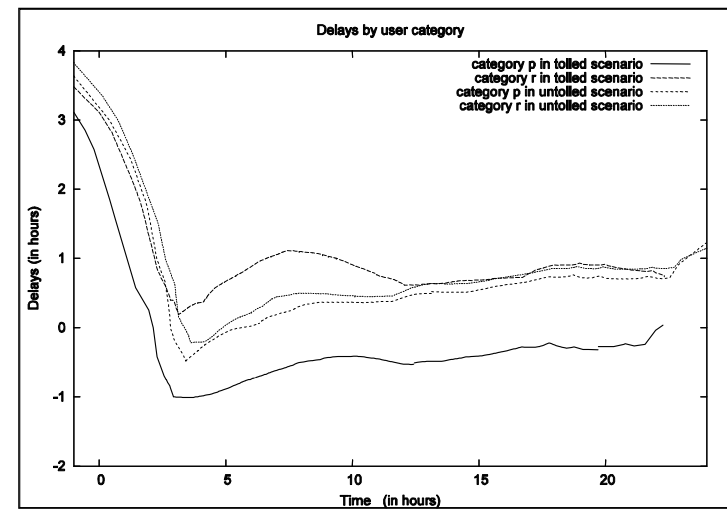
The distributions of desired arrival instants for each user class are also taken from Sautter (2007) and are based on historical data from the road operator Cofiroute, in charge of the SR91 since 1995. The capacities of two routes are set to 8000 pcu/hour for the free route, and 2500 pcu/hour for the two additional lanes.

Scenarios

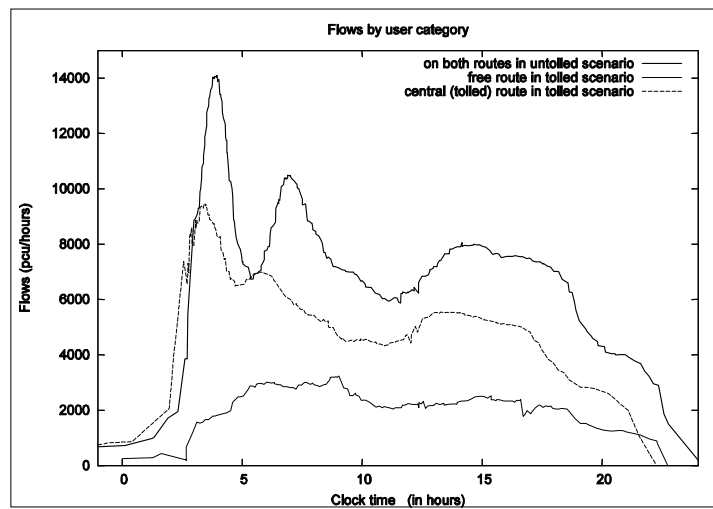
Two scenarios have been simulated. The first is called *untolled*. The amount of the toll fare on the two additional lanes was set to 0, and the user equilibrium with departure time choice has been computed using the LTK. The second scenario is called *tolled*. The time-varying toll fare was set equal to the curve plotted in Fig.2(d). A comparative study allows us to discuss the net effect of tolling.



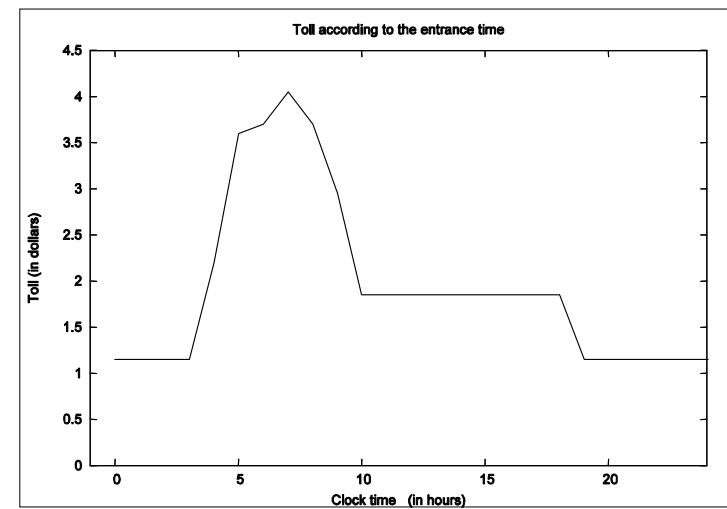
(a)



(b)



(c)



(d)

Figure 2 : Equilibrium values of the SR91 under two scenarios.

Results

The results of each route and each scenario are plotted in Fig.2(a-c) . In both cases the results are consistent with the bottleneck model. In the untolled scenario, the travel time pattern shows a double peak with slopes corresponding to the one induced by the ratios γ/α and β/α . The travel time maxima are obtained when delays are close to 0. Users are using indifferently the two routes and the travel time is rigorously the same. In the tolled scenario, the two users types are segregated by the toll ; only users of the category r uses the tolled route.

The travel time pattern is affected by the time-varying toll. Let us note that the tolling scheme globally reduces travel times. This is achieved by two mechanisms. On the tolled route the toll tends to spread the traffic, thus reducing congestion. On the untolled route, the reduction is less pronounced and is due to a small reduction in traffic resulting of the departure of the users of category r .

Table 1 exemplifies the results of an hypothetical socio-economic analysis based on the results of the simulation. The benefit of the toll is driven by the time savings made by the class r , which is natural as they have the entire benefit of the faster tolled lanes. Yet the increase in traversal costs of class p is compensated by a decrease in delay costs. This is not straightforward but a closer inspection reveals that as class r uses only very marginally the free route in the tolled scenario, the class p has relatively more capacity than in the untolled scenario. However as there is no toll on their route they tend to schedule their trips regarding to congestion costs, so the benefit of this additional capacity results mainly in a decrease in delay cost.

Category	Untolled		Tolled		Difference	
	p	r	P	r	p	r
Traversal Costs	2947	2525	3000	2432	-53	93
Delay Costs	1873	2923	1823	2911	50	12
Total/Category	4820	5448	4823	5343	-3	105
Total/Scenarii	10268		10166		102	

Table 1: Results from a hypothetical socio-economic analysis (values are in M\$).

4.2. The VDR example

During summer holydays, a significant part of the trans-european road traffic is concentrated in the Vallée du Rhône (VDR) area. Tourists coming from northern Europe (including Belgium, the Netherlands, Germany and Great Britain), travel across France to reach (or return from) southern countries (e.g Italy and Spain), meeting on their way people from the Paris area. The situation is depicted in Fig.2 The map on the left hand side (Fig.2(a)) shows the location of the VDR area and the structure of traffic flows from foreign countries. The network of main roads in the French network is mapped in Fig.2(b), along with the set of o-d pairs this example is concerned with. The main axis in the VDR area is the A7 motorway, located between Lyon (LY in Fig.2(b)) and Orange (OR in Fig.2(b)). The distance between those two cities is around 200km.

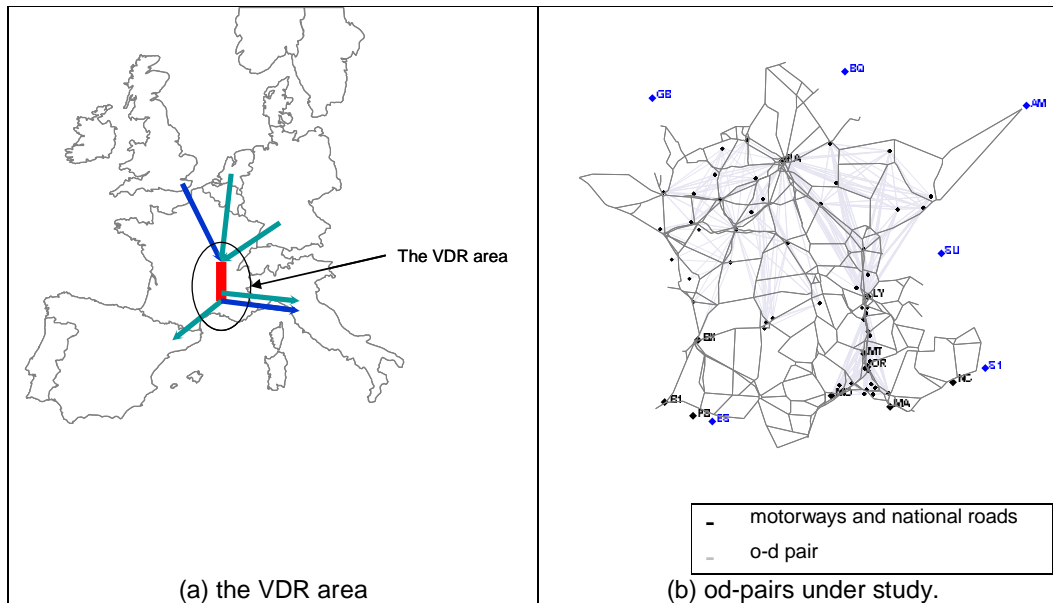


Figure 2: The major road network in France.

During summer Saturdays, traffic conditions on motorways are usually very bad, especially on the A7, because of high levels of congestion. To better operate the network, motorway operators may be interested in studying time varying tolling strategies. Results presented in the sequel illustrate the ability of our model to handle such kind of studies on large networks.

Input data

Most of the data was provided to us by courtesy of companies of the Vinci Group (ASF, APRR and Cofiroute) operating the motorway network in the area of interest. The network comprises 2404 arcs and 939 nodes. We knew for each arc its capacity, free flow travel time for passenger cars, and travel price for passenger cars. The demand was expressed for 628 o-d pairs. The simulated day was July the 14th, 2007. Two user classes, named v_l and v_{ls} , were considered. Those two classes correspond to passenger cars¹. They share most of their characteristics (same free flow travel time, same toll prices, etc). They are distinguished only by their delay cost functions, plotted in Fig.3(a). For user class v_l , the penalty of arriving later than scheduled grows linearly at a very high rate, while the penalty of arriving sooner grows at a lower rate. The delay cost function of user class v_{ls} is a little bit more complex. Around 0, it has a classical V-shape, except that the cost of a early arrival grows faster than the cost of a late arrival. Between 6 and 18 the cost of the delay is infinite. Around 24, the shape of the delay cost function is similar than around 0, except that it is shifted up by an amount that correspond to the cost of rescheduling the departure to the day after. The values of the delay cost evolve similarly around 24. This expresses the cost of rescheduling the departure to the day before.

¹ The traffic of heavy vehicles is ignored since traffic regulation rules forbid truck traffic during some of the most congested days in summer.

We were provided with a departure choice demand matrix $X_{I,U}^*$. For each user class u , o-d pair i and arrival instant h^* , $X_{I,U}^*$ gives the density $x_{i,u}^*(h^*)$ of users wishing to minimize their total traversal cost.

The number of user classes, the user cost functions and the departure choice demand matrix were provide as such. They have been inferred for the simulated day from a large survey conducted by the motorway operators in year 2008.

The traversal cost of an arc a for a user of class u entering the arc at an instant h is a linear combination of the monetary price $p_{a,u}$ and of the arc traversal time $t_{a,u}$. It is given by:

$$c_{a,u}(h) = p_{a,u}(h) + \alpha_u t_{a,u}(h)$$

where α_u is the value of time for the user class u . The user classes v_l and v_{ls} are such that α_{v_l} is by far lower than $\alpha_{v_{ls}}$.

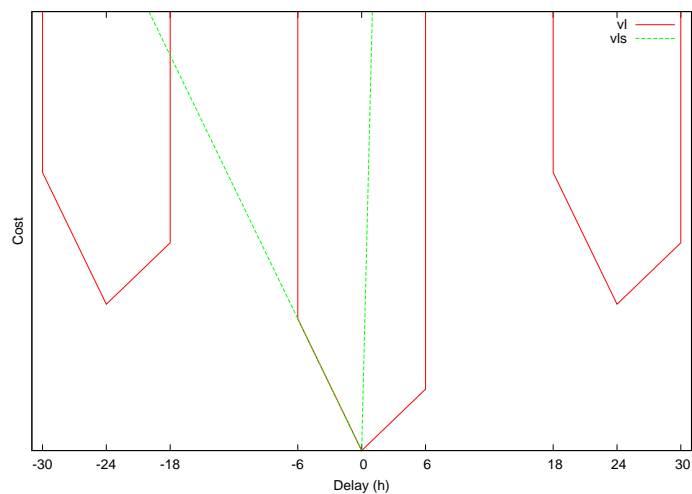
The monetary price $p_{a,u}$ includes the amount of the toll fare. The existing toll fares were made time dependant by multiplying them by a time dependant factor, which is plotted in Fig.3(b). This factor is greater than one (i.e. the fare is higher than usual) between 5 and 17. Its is lower than 1 between 20 and 34, meaning that the amount of the fare is lower than usual between 8 p.m of the simulated day and 10 a.m of the day after.

Calibration

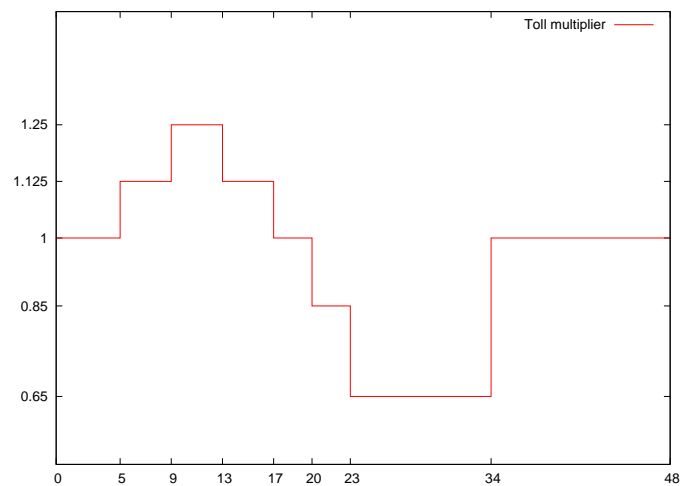
Most of the motorways of the network under study are equipped with closed toll systems. As a consequence, we had at our disposal an accurate time-dependant o-d matrix for the simulated day, built from the toll stations records. This allowed for a fine grain calibration of the model, by adjusting its parameters until simulated traffic flows computed by a (fixed demand) traffic assignment match well traffic counts data, for a significant percentage of motorway sections.

Results

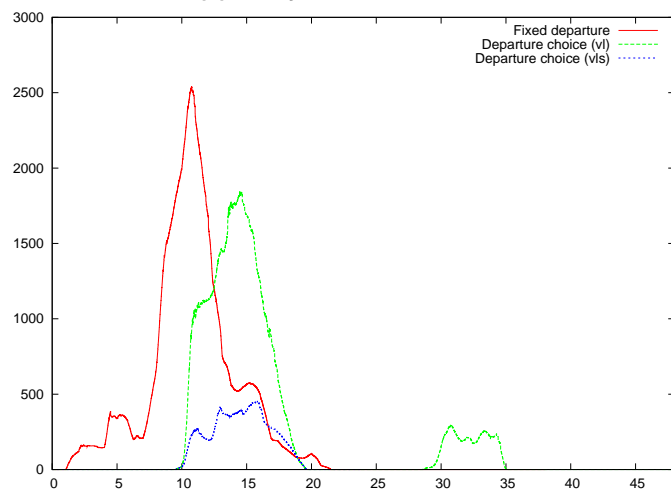
Once the model calibrated, we could run a user optimum assignment with departure time choice and time-varying tolls. Results were compared to the fixed-demand, constant-toll assignment. The difference between the fixed-demand and the computed demand is synthesized in Fig.3(d). The two plots are the sum of the cumulative flows at departure, for all o-d pairs. The variations of the demand computed with departure time choice clearly indicates that a significant part of users have rescheduled their departure day to the day after. This is confirmed when observing the evolution of the pattern of congestion during the simulated day, Fig.3(c). The plotted value is an estimate of the total time spent in congestion: at a given instant during the day, the input flow rate of each arc is multiplied by the time spent in queue on this arc; the sum for all arcs is plotted. In the fixed-demand scenario, congestion starts soon after 0 a.m. and vanishes around 9 p.m, with a significant peak between 9 a.m and 12. With departure time choice, a fraction of the demand has rescheduled its departure to the day after. This leads to a congestion period that last half longer than in the fixed demand scenario.



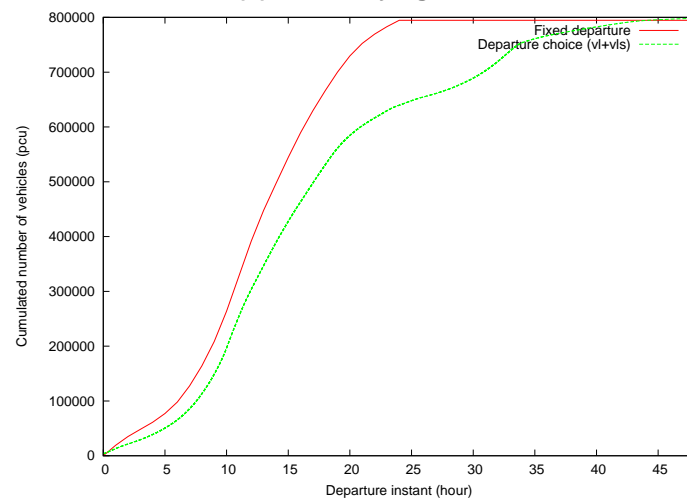
(a) Delay cost functions.



(b) Time varying tool.



(c) Extra time spent.



(d) Demand.

Figure 3: Inputs and outputs for the VDR example.

CONCLUSION

We have presented a model for dynamic traffic assignment with departure choice in interurban networks. While most of the models concerned with departure choice modelling are designed with commuting patterns in mind, it has been shown that the requirements for an appropriate modelling of interurban trips departure choice are substantially different. Using a continuous, rather than discrete, distribution of preferred arrival instants seems to better reflect the heterogeneity of users' preferences at arrival. The convexity of delay cost functions has to be relaxed, since it does not allow for some desired behaviours (e.g. day to day departure rescheduling).

Semi empirical algorithms have been proposed, implemented, and illustrated by two numerical examples. The results are encouraging and show some of the benefits practitioners can take from using our approach when studying congestion management schemes. A substantial amount of work is still needed to properly state the appropriate numerical solution techniques.

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Notes

¹LADTA stands for Lumped Analytical Dynamic Traffic Assignment.

²LTK stands for the LADTA ToolKit.

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